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Quantitative analysis of the linear optical character of the anterior segment of the eye*

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An important issue in the quantitative analysis of optical systems is, for example, the question of how to calculate an average of a set of eyes. An average that also has an optical character that is representative or central to the optical characters of the eyes within that set of eyes. In the case of refraction, an average power is readily calculated as the arithmetic average of several dioptric power matrices. The *exponential-mean-log-transference* has been proposed by Harris as the most promising solution to the question of the average eye. For such an average to be useful, it is necessary that the exponential-mean-log-transference satisfies conditions of existence, uniqueness and symplecticity. The first-order optical nature of a centred optical system (or eye) is completely characterized by the 4x4 ray transference. The augmented ray transference can be represented as a 5x5 matrix and is usually partitioned into 2x2 and 2x1 submatrices. They are the dilation **A**, disjugacy, **B** divergence **C,** divarication **D** transverse translation **e** and deflection π . These are the six fundamental first-order optical properties of the system. Other optical properties, called derived properties, of the system can be obtained from them. Excluding decentred or tilted elements, everything that can happen to a ray is described by a 4x4 system matrix. The transference, then, defines the four **A**, **B**, **C** and **D** fundamental optical properties of the system.

The thesis presents the first quantitative and multivariate analysis of the linear-optical character of the cornea and anterior segment of the eye and investigates its nature and characteristics using concepts such as exp-mean-log-transferences. To determine the transferences of the cornea or anterior segment of the eye one needs measurements of three specific elements of the cornea (namely, central powers, or radii, of the anterior and posterior corneal surfaces and the axial corneal thickness) and also the axial or central depth of the anterior chamber. The refractive indices of the cornea and the aqueous humour were taken as 1.376 and 1.336, respectively. The various multivariate methods of analysis used for this study are applied for the first time for an investigation of anterior eye transferences. These methods allow for a much more detailed and complete understanding of several issues such as the average eye of a set of eyes and completely new ideas in optometry and ophthalmology such as variance of optical systems in a 10-dimensional Hamiltonian space.

Subjects had to fulfill certain selection criteria to be included in this study; the visual acuity had to be 6/6 or better (right eye only), and subjects with any observable ocular disease, history of ocular trauma or refractive surgery were not accepted. Consecutive measurements of the central anterior and posterior corneal keratometry, corneal thickness and the axial anterior chamber depth were obtained with the Oculus Pentacam. A minimum of 40 successive measurements

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of the anterior and posterior central corneal surface powers, central corneal thickness and axial anterior chamber depths were obtained over a short period of time, with refocusing between each measurement for the right eye of each subject. These measurements were transformed from the conventional notation (that is sphere power, cylinder power and cylinder axis) to the scientific notation (stigmatic, ortho-antistigmatic and oblique antistigmatic components of power, that is, $F_{\mathrm{I}}\mathbf{I}$, $F_{\mathrm{J}}\mathbf{J}$ and $F_{\mathrm{K}}\mathbf{K}$ respectively).

From the measurements of the central radii of the anterior and posterior corneal surfaces, central corneal thickness and axial anterior chamber depths, 4x4 ray transferences for both the cornea and anterior segment of the eye were calculated. The principal matrix logarithms for all the transferences were determined. This produced samples of at least 40 transformed transferences each for the cornea and the anterior segment of the eye. They are each represented by 40 points in a 10-dimensional Hamiltonian space. Means and 10x10 variance-covariance matrices each for the cornea and the anterior segment were calculated from the transformed transferences.

The matrix exponential of the means gave values for the mean transferences of the cornea and the anterior segment, and they represented the average cornea and the average anterior segment of the eye concerned. From the average transformed transferences of the cornea and anterior segment of the eye, vector fields were plotted to show how the light rays traverse the cornea and the anterior segment of the eye given the defined optical systems. That is, the transference converts the incident ray vector into an emergent ray vector. The process is treated as linear and is therefore the basis of linear optics.

Summary of important findings or contributions of this study

1. Short-term intraocular variation of the anterior and posterior autokeratometric measurements and non-cycloplegic autorefraction were determined. Generally there was less variation of the posterior corneal keratometry than that for the anterior surface. The anterior corneal mean keratometry obtained with the Pentacam varied from 42 to 49 D depending on the subject while the posterior corneal keratometry varied from –5.77 to –6.19 D.
2. Variation of the coordinate vectors of the log transferences has been indicated via 10x10 variance-covariance matrices in log-transference space. These variances were generally of small magnitude for the eyes concerned and this applies to both the optical system considered as a cornea or as an anterior segment. However, some covariances were of large magnitude.
3. Meaningful averages ($\tilde{\mathbf{T}}$) of ray transferences ($\mathbf{T}$) were computed from the log or transformed transferences.
4. Ray diagrams or ray vector fields were plotted to partially explain the influences of the corneal and anterior segment ray transferences on rays. Such methods are likely to become more useful and common as further work in this field of study occurs.
5. Many of the graphical and numerical approaches in this thesis will likely become important in several areas of optometry, ophthalmology and vision science to study issues relating to the linear optics of the eye or parts of the eye (such as the cornea, anterior segment, crystalline lens or whole eye), especially as they might be affected by influences of various diseases and physiological processes involving the eye.