RESEARCH NOTE

Dark refraction shift with allowance for astigmatism

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Abstract

Purpose: To show that the dark refraction shift (dark focus) is a more complicated phenomenon than implied when presented as spherical.

Methods: Fifty autorefractor measurements of refractive state of the right eye were obtained in light and dark conditions. Multivariate methods were used to analyze the data and stereo-pair scatter plots, polar meridional profiles and other means of presenting results are used to show important characteristics of the dark refraction shift.

Results: The complexity of the dark refraction shift is indicated by stereo-pair scatter plots showing the amount of stigmatic and antistigmatic variation that occurs in light and dark conditions. The mean dark refraction shift is presented in a complete manner including all three components of refractive state. The greater variance and covariance under dark conditions is clearly shown by the term-by-term dark-light variance-covariance ratio and polar profiles of variance and covariance.

Conclusions: The dark refraction shift is a more complicated phenomenon than implied by representations as purely spherical in nature.

Keywords: dark refraction shift, dark focus, multivariate methods, stigmatic, antistigmatic, polar profiles.

The refractive state of the human eye is rarely purely stigmatic (spherical) in nature. Accommodation can induce astig-

matic change in refractive state¹. However, as far as we are aware, studies investigating the dark focus report findings in which the dark focus is recorded as spherical only. In many instances the technique used to measure the dark focus is limited to spherical measurements²⁻¹³, in other instances infrared optometers are used but only the nearest equivalent sphere is recorded¹⁴⁻¹⁷, and in other studies no mention is made of how the infra-red optometer measurements are used to produce spherical-only recordings¹⁸⁻²⁴. In some studies the researchers deliberately disable the axis rotation mechanism of the optometer and use a single, usually the vertical, meridian to determine the dark focus²⁵⁻²⁸. Apart from the use of standard deviation and standard error of the mean^{4-8, 10-11, 14-15, 20-22}, no other approach is used to evaluate the characteristics of the variation of multiple measurements of refractive state under dark conditions.

We prefer the term dark refraction shift to the traditional term dark focus. Formally we define the dark refraction shift to be $\triangle F$ where $\triangle F = F_D - F_L$.

 F_D and F_L are dioptric power matrices and represent refraction in the dark and light respectively. We believe that aspects of the dark refraction shift have been missed in the past because incomplete, purely spherical measures of the dark refraction shift have been reported. They include: the extent of stigmatic and antistig-



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matic variation, a complete measure of variance and covariance of refraction in the dark, the relationship between light and dark condition variance and covariance as well as the changes in the characteristics of variation occurring in light and dark conditions. The purpose of this note is to illustrate these phenomena.

Method

Twenty individuals acted as subjects in this study, 10 were aged between 21 and 35 years of age (pre-presbyopes) and the other 10 were aged between 40 and 65 years of age (presbyopes). The data from only one subject (aged 21) is used in this article in the interests of brevity. This subject was selected because her findings highlighted interesting aspects of the data. All subjects were treated according to the tenets of the Declaration of Helsinki and all subjects were volunteers and gave informed consent after the research protocol had been explained to them. All subjects were selected after undergoing a battery of screening tests. A Hoya AR 550 autorefractor was used to take 50 measurements of refractive state of the right eye under two conditions, the light condition where the instrument target and the room environment were fully illuminated and the dark condition where the instrument target and room illumination were largely eliminated. Each subject spent five minutes in darkness before the dark condition measurements were taken. The autorefractor was set to



Figure 1. (a) Stereo-pair showing light (red) and dark (green) condition measurements. Included are the 95% distribution ellipsoids. The origin is [0 0 0] D and tick intervals are 0.25**I**, 0.25**J** and 0.25**K** D. The difference in size and positioning of the two clusters show the changes in refractive state and its variation under the two stimulus conditions. measure refractive state in steps of 0.01 D and the vertex distance was set at zero.

Power is represented as the symmetric dioptric power matrix

$$\mathbf{F} = F_{\text{st}} \mathbf{I} \ F_{\text{or}} \mathbf{J} \ F_{\text{ob}} \mathbf{K}$$

where $\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \ \mathbf{J} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ and $\mathbf{K} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

 $F_{\rm st}$, $F_{\rm or}$ and $F_{\rm ob}$ are the stigmatic, ortho-antistigmatic and oblique antistigmatic coefficients of the power respectively. On the face of it $F_{\rm st}$, $F_{\rm or}$ and $F_{\rm ob}$ are equivalent to M, J_0 and J_{45} respectively defined by Thibos and coworkers²⁹. However, the analysis here has a different basis; in contrast to their power vectors it is in terms of dioptric power matrices. Graphical representations of the matrices in symmetric dioptric power space are constructed using axes representing scalar multiples of **I**, **J** and **K**. Means are obtained as the arithmetic mean of the matrices. Variances and covariances are calculated for $F_{\rm st}$, $F_{\rm or}$ and $F_{\rm ob}$. All data collected were analyzed using methods developed by Harris³⁰⁻³⁵ and software developed by Malan³⁶, Harris and Rubin³⁷.

Results

Table 1 gives the mean refractive state (of 50 autorefractor measurements) for the light and dark condition in conventional spherocylindrical terms as well as in terms of the stigmatic (F_{st}) and the ortho- (F_{or}) and oblique (F_{ob}) anti-



Figure 1. (b) Scatter plot showing the data viewed along the stigmatic axis. Only a single set of axes are presented showing light (red) and dark (green) condition data. The orientations of the major axes of the two ellipsoids are noticeably different, indicating the change in antistigmatic variation in response to the stimulus conditions.

 Table 1.
 Mean refractive state for light and dark conditions and mean dark refraction shift given in complete conventional and component notation.

 (* The ortho-antistigmatic mean seems incorrect but round-off has resulted in this apparent discrepance)

this uppurent discrepancy).										
	Sph	Cyl	Axis	F _{st}	For	F _{ob}				
Light	0.57	- 0.35	29	0.40	0.09	0.15				
Dark	0.02	- 0.39	29	-0.17	0.11	0.16				
Mean dark refration shift	- 0.55	- 0.04	20	- 0.57	0.01*	0.01				

Table 2. Variances and covariances in D^2 of the coefficients of power; s_{11} , s_{22} , and s_{33} represent the stigmatic (F_{st}), the ortho-antistigmatic (F_{or}) and the oblique antistigmatic (F_{ob}) coefficients respectively. s_{12} , s_{13} and s_{23} represent the stigmatic-ortho-antistigmatic, stigmatic-oblique antistigmatic and ortho-oblique antistigmatic covariances respectively.

Condition	<i>s</i> ₁₁	<i>s</i> ₂₂	<i>s</i> ₃₃	<i>s</i> ₁₂	<i>s</i> ₁₃	s ₂₃
Light	0.0116	0.0037	0.0016	- 0.0029	- 0.0006	0.0010
Dark	0.0420	0.0063	0.0147	- 0.0010	- 0.0084	0.0036

Table 3. The term-by-term dark-light variance-covariance ratios are shown. r_{11} , r_{22} , and r_{33} indicate the ratio of the stigmatic, ortho-antistigmatic and oblique antistigmatic variance components respectively. r_{12} , r_{13} and r_{23} indicate the stigmatic-ortho-antistigmatic, stigmatic-oblique antistigmatic and ortho-oblique anstigmatic covariance ratios respectively.



Figure 2 (a). Polar meridional profiles of curvital (larger, outer profile) and torsional (smaller, inner profile) variance are shown. (2a) Light condition. The meridian of maximum curvital variance is close to the vertical meridian.



(b) Dark condition. The meridian of maximum curvital variance is approximately 1300. The scale in (b) is twice that in (a).

stigmatic coefficients. Also included in Table 1 is the mean dark refraction shift.Figure 1a is a stereo-pair showing 50 measurements of refractive state taken in the light (red) and dark (green). Included for each cluster of measurements are the 95% ellipsoidal surfaces of constant probability density (distribution ellipsoids). The origin of the axes is [0 0 0] D and the tick intervals are 0.25I D, 0.25J D and 0.25K D.

The stereo-pair can be fused by converging the eyes to a point in front of the page, this way the three-dimensional character of the data can be appreciated.

Important aspects of Figure 1a to note are each ellipsoid is elongated parallel, or almost parallel, to the stigmatic axis showing that mainly stigmatic (spherical) variation took place during the measurements. The dark cluster (green) is larger than the light cluster (red) showing the increased variation taking place in the dark, in both the stigmatic and antistigmatic direction (indicated by the larger waist of the green ellipsoid). The dark refraction shift is indicated by the lower positioning of the green cluster (in this orientation) relative to the red cluster.

Table 2 gives the vectorized variance and covariance matrices for the light and dark measurements, units are D^2 . Table 2 gives the only complete representation of the variance and covariance that exists between the components of refractive state. An important indicator of the difference in variance and covariance that occurred between light and dark measurements is the term-by-term dark-light variance-covariance ratio shown in Table 3.

The important aspects of Table 3 are: the stigmatic variance ratio, 3.6, and the oblique variance ratio, 9.2. The stigmatic ratio shows that there



was about 3.6 times more stigmatic or spherical, variation occurring in measurements taken in the dark compared with those taken in the light (seen as the greater stigmatic elongation of the green cluster in Figure 1a) while the oblique antistigmatic ratio shows about nine times more variation in the dark indicating the increased antistigmatic variation (seen as the larger waist of the green ellipsoid in Figure 1a). The increased oblique antistigmatic variation is seen in Figure 1b and is shown by the larger green ellipsoid that is elongated roughly parallel to the oblique antistigmatic, K, axis. Figure 2 shows polar meridional profiles of variance for the light (Figure 2a) and dark (Figure 2b) data. The outer, larger profiles indicate the curvital variance across meridians and the smaller, inner profile the torsional variance across meridians of the eye. Figure 2a has the largest curvital variance close to the vertical meridian, a phenomenon that has been seen previously and is thought to be due to the blink process. However, Figure 2b shows the greatest curvital variance close to the 130° meridian.

The scale is also larger in Figure 2b, indicating the greater amount of variance occurring under dark conditions, and is also shown in Figure 1a and Tables 2 and 3. It is not clear why the meridian showing the greatest variance should change under light and dark conditions.

Discussion

Presenting the results of research²⁻²⁸ involving the dark refraction shift in terms of only a spherical component of refractive state is incomplete; such results give no indication of the complexity of the phenomenon. This article has presented the dark refraction shift of a single individual and has emphasized aspects of the phenomenon other than only the spherical component. In particular it shows that the dark refraction shift has an antistigmatic component, that variation occurs in both the stigmatic as well as antistigmatic components and that stigmatic and antistigmatic variation increase under dark conditions.

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