Sensitivity of the corneal-plane refractive compensation to change in power and axial position of an intraocular lens

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Abstract

If an intraocular lens is displaced or if its power is changed what are the consequences for the refractive compensation of the eye? Gaussian optics is used to obtain explicit formulae for the sensitivity of the corneal-plane refractive compensation (also called the refraction, refractive state, etc) to change in power and axial displacement of a thin intraocular lens implanted in a simple eye. In particular, for a pseudophakic Gullstrand simplified eye with intraocular lens placed 5 mm behind the cornea the sensitivity to errors in the power of the intraocular lens is about −0.71 for an intraocular lens of power 20 D, that is, the refractive compensation decreases by about 0.71 dioptres per dioptre increase in the power of the intraocular lens. More generally the sensitivity is approximately $-0.63 - (0.0037\text{m})F_\text{I}$ where F_I is the power of the intraocular lens. Also for Gullstrand's simplified eye the sensitivity of refractive compensation to axial displacement of the intraocular lens is approximately linear in F_I, about (64D) F_I, in fact. That is, for each dioptre of the power of the intraocular lens the refractive compensation increases by about 0.064 dioptres per millimetre of axial displacement towards the retina.

Key words: intraocular lens, sensitivity, axial displacement, Gaussian optics, refraction, refractive compensation, Gullstrand's simplified eye.

Introduction

How important is the positioning and the power of an intraocular lens? If the lens becomes displaced or the incorrect power is used what is the consequence for the refractive compensation of that eye? An accompanying paper[1] describes a general procedure for calculating the sensitivity of an optical property of an eye to change or error within the eye. The purpose of this paper is to apply the method to an eye containing an intraocular lens. More specifically the paper calculates the sensitivity of the corneal-plane refractive compensation of the eye to change in power and axial position of the intraocular lens. In order to display the ideas, minimally obscured by mathematics, use is made of the simplest reasonable optics (Gaussian optics) and the simplest pseudophakic eye (a single refracting surface for the cornea and a thin intraocular lens). For detail on the linear optics of the pseudophakic eye, not necessarily thin, and on the effect of axial position of the intraocular lens the reader is referred elsewhere[2, 3].

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Pseudophakic simplified eye

Consider a pseudophakic simplified eye. The eye can be regarded as consisting of two systems, the anterior part S_A and the posterior part S_P (Figure 1). The anterior part has entrance plane T_0 immediately in front of the cornea K and exit plane T_I immediately in front of the intraocular lens I. It consists of a thin system (the cornea) of power F_K followed by a homogeneous gap of reduced width ζ_A. The posterior part has entrance plane T_I and exit plane T immediately in front of the retina R. Its structure is optically similar: it also consists of a thin system (an intraocular lens I) of power F_I and a homogeneous gap of reduced width ζ_P. The index of refraction is n within the eye and 1 in front of the eye.
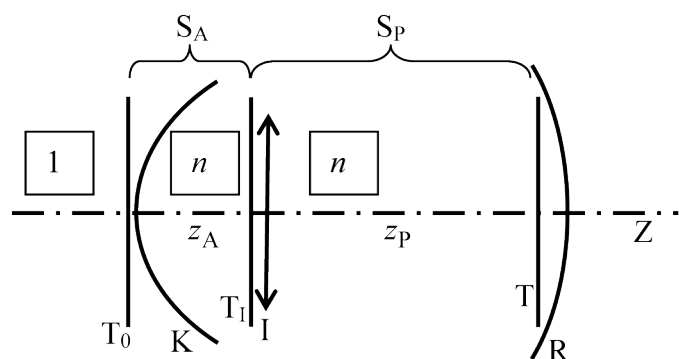

Figure 1 A simplified eye with cornea K of power F_K and an implanted thin intraocular lens I of power F_I. The distance between the cornea and the intraocular lens is z_A and between I and the retina R is z_P. S_A is the anterior part and S_P the posterior part of the eye.

Each part of the eye has a transference of the form given by Equation 19 of the accompanying paper[1]; for the anterior part

$$\mathbf{S}_A = \begin{pmatrix} 1 - \zeta_A F_K & \zeta_A \\ -F_K & 1 \end{pmatrix} \tag{1}$$

and for the posterior part

$$\mathbf{S}_P = \begin{pmatrix} 1 - \zeta_P F_I & \zeta_P \\ -F_I & 1 \end{pmatrix}. \tag{2}$$

It follows from Equation 11 of that paper that the transference of the pseudophakic eye is

$$\mathbf{S} = \begin{pmatrix} 1 - \zeta_P F_I & \zeta_P \\ -F_I & 1 \end{pmatrix} \begin{pmatrix} 1 - \zeta_A F_K & \zeta_A \\ -F_K & 1 \end{pmatrix} \tag{3}$$

which multiplies out to

$$\mathbf{S} = \begin{pmatrix} (1 - \zeta_P F_I)(1 - \zeta_A F_K) - \zeta_P F_K & (1 - \zeta_P F_I)\zeta_A + \zeta_P \\ \bullet & \bullet \end{pmatrix}. \tag{4}$$

The bottom row of the matrix is not needed below and is omitted.

Sensitivity of the refractive compensation to change in power of the intraocular lens

Applying Equation 4 above to Equation 16 of the previous paper[1] one obtains the corneal-plane refractive compensation

$$F_0 = \frac{1 - \zeta_P F_I}{\zeta_A + \zeta_P - \zeta_A \zeta_P F_I} - F_K. \tag{5}$$

(The denominator, with form reminiscent of the form of what is commonly called the equivalent power of a thick lens, might be regarded as an equivalent or effective reduced length of the eye.) Equation 5 defines F_0 as a function $F_0^{F_K \zeta_A F_I \zeta_P}$ of the four independent variables F_K, ζ_A, F_I and ζ_P. It also implies four derivatives, that is, four sensitivities. One of them is obvious: the sensitivity of the compensation to change in corneal power, that is, $F_0^{F_K' \zeta_A F_I \zeta_P}$, is -1, in agreement with what we would expect.

We are interested in the sensitivity of the corneal-plane refractive compensation to change in power of the intraocular lens, that is $F_0^{F_K \zeta_A F_I' \zeta_P}$. Differentiation of Equation 5 results in

$$F_0^{F_K \zeta_A F_I' \zeta_P} = -1 / (1 + \zeta_A / \zeta_P - \zeta_A F_I)^2. \tag{6}$$

This shows in particular that the sensitivity is independent of the corneal power F_K.

Sensitivity of the refractive compensation to axial displacement of the intraocular lens

Suppose instead one is interested in the sensitivity of the corneal-plane refractive compensation to

change in axial position of the intraocular lens. We need to work in terms of distances instead of reduced distances. Equation 5 can be expressed as

$$F_0 = \frac{n - z_P F_I}{z_A + z_P - z_A z_P F_I / n} - F_K \; . \qquad (7)$$

z_A and z_P are the widths of the anterior and posterior parts of the eye respectively.

Axial displacement of the lens changes both z_A and z_P. One needs to find an independent variable that can be changed while all others remain fixed. An obvious procedure is to define z the length of the eye:

$$z = z_A + z_P \qquad (8)$$

Substituting $z - z_A$ for z_P in Equation 7 and rearranging we obtain

$$F_0 = \frac{n - (z - z_A)F_I}{z - (z - z_A)z_A F_I / n} - F_K \; . \qquad (9)$$

This represents the function $F_0^{F_K z_A z n F_I}$ of the five independent variables z_A, F_K, F_I, z and n and each of the five derivatives represent sensitivities.

In order to find the sensitivity of the corneal-plane refractive compensation to change in distance z_A with F_K, F_I, z and n constant we need to differentiate with respect to z_A. The result turns out to be

$$F_0^{F_K z_A' z n F_I} = \frac{n(z - z_A)(2n - (z - z_A)F_I)F_I}{(nz - z z_A F_I + z_A^2 F_I)^2} \; , \qquad (10)$$

a sensitivity which, as expected, is also independent of the corneal power.

Application to a pseudophakic simplified eye

Consider Gullstrand's simplified eye from which the lens has been removed. The index of the aqueous and vitreous humors is $n = 1.336$, the length of the eye is $z = 24$ mm and the power of the cornea is $F_K = 48.03$D.[4]

Suppose a thin intraocular lens is implanted at a distance of $z_A = 5$ mm behind the cornea. Hence $z_P = 19$ mm and the reduced widths are $\zeta_A = 3.74$ mm and $\zeta_P = 14.22$ mm. The corneal-plane refractive compensation is given by Equations 5, 7 or 9; for an implant of power $F_I = 20$ D the corneal-plane refractive compensation turns out to be −0.74 D.

Equation 6 results in

$$F_0^{F_K \zeta_A F_I' \zeta_P} = -1/(1.263 - (0.00374 \, \text{m})F_I)^2 \; . \qquad (11)$$

Because the coefficient of F_I in the denominator is in metres F_I is necessarily in dioptres. For $F_I = 20$ D this gives a sensitivity of about −0.71. One can think of this as −0.71 D/D. Thus for 1 D increase in power of the intraocular lens the corneal-plane refractive compensation will decrease by about 0.71 D, thus changing the compensation to an estimated value of about −1.45 D. (This agrees with the true compensation calculated via Equation 5 for $F_I = 21$ D.) A decrease of 1D results very nearly in emmetropia. For $F_I = 10$D the sensitivity is −0.67 and for $F_I = 30$ D it is −0.75. The dependence of sensitivity on the power of the intraocular lens is roughly affine, the binomial expansion being

$$F_0^{F_K \zeta_A F_I' \zeta_P} \approx -0.63 - (0.0037 \, \text{m})F_I \; . \qquad (12)$$

From Equation 10 we obtain the sensitivity of the refractive compensation to change in axial position. For $F_I = 20$ D the sensitivity is about 1280 D² or about 1.28 D/mm. An axial shift towards the retina of 1 mm will increase the refractive compensation by about 1.28 D to an estimated compensation of about D. The actual refractive compensation, calculated via Equation 5, is 0.52 D. Hence the error in the estimate is 0.02 D. For $F_I = 10$ D the sensitivity is about 650 D² or 0.65 D/mm and for $F_I = 30$ D it is about 1880 D². In fact the sensitivity is approximately linear in F_I:

$$F_0^{F_K z_A' z n F_I} \approx (64 \, \text{D})F_I \; . \qquad (13)$$

Thus for every dioptre of power of the intraocular lens the corneal-plane refractive compensation increases
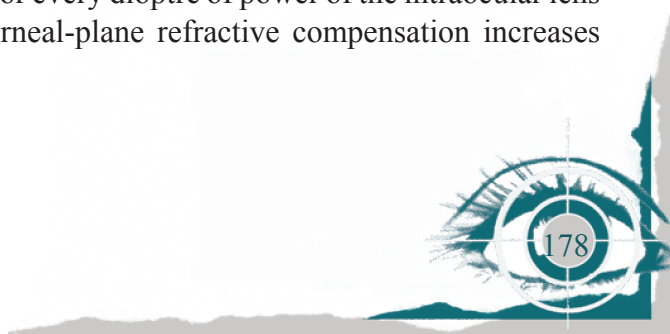
by about 0.064 D per millimetre of axial displacement towards the retina.

Concluding remarks

Explicit formulae have been derived here, using Gaussian optics, for a pseudophakic eye derived from Gullstrand's simplified eye. The formulae give the sensitivity of the corneal-plane refractive compensation to error or change in axial position (Equation 10) and power (Equation 5) of an intraocular lens. Sensitivity to change or error in power is a relatively-weak and approximately-affine function of the power of the intraocular lens (Equation 12) whereas sensitivity to axial displacement is approximately linear in the power (Equation 13).

Acknowledgements

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