Yves Le Grand and the dioptric power matrix

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In a previously published paper, the present author asserted that Yves Le Grand came very close to describing the dioptric power matrix in 1945 and that, in particular, he presented expressions for the entries of the matrix. The purpose of this note is to weaken that assertion and to argue that, while he certainly did describe some features of the matrix, his expression for the off-diagonal entries of the matrix for thin systems was incorrect (a missing factor appears not to have been a typographical error) and that, although he gave basic formulae of Gaussian optics involving dioptric power, he gave no hint of natural and powerful generalisations for linear optics involving the dioptric power matrix.

In an analysis[1] of an appendix[2] to Le Grand's 1945 book,[3] I argued that, although he did not describe the dioptric power matrix as such, he certainly came very close to doing so. It seems to me now, however, that this conclusion was too strong. The purpose of this note is to argue for a weaker conclusion.

Let $\mathbf{S}$ be the ray transference of an optical system. It is a real matrix, 2×2 in Gaussian optics and 4×4 in linear optics. The bottom-left entry in the 2×2 case is the negative of the dioptric power F of the system, and the bottom-left 2×2 block in the 4×4 case is the dioptric power matrix

$$\mathbf{F}=\begin{pmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{pmatrix}. \qquad \text{[Eqn 1]}$$

$\mathbf{F}$ is symmetric ($f_{21}=f_{12}$) for thin systems but general otherwise. (References were given before[1].)

Analysis[1] shows that Le Grand's matrix **M** is not the transference **S** but its inverse $\mathbf{S}^{-1}$, although he does not use that terminology, and he does not view **M** as an operator relating the incident and emergent states of a ray traversing a system. For the power F, Le Grand initially uses the symbol a and then later the symbol D; he calls F the *true power* of the system but changes the term to *equivalent power* in the third edition of his book[4] that appeared in 1964. On the other hand, he does not use a symbol for $\mathbf{F}$ at all, nor does he assign it a name. Instead, he talks of three (separate) powers in the case of a thin system; two are equivalent to f_{11} and f_{22}; the third (Le Grand's A) is not f_{21} (or, equivalently, f_{12}) but double it (see Footnote 53 of Reference 1). Multiplying Le Grand's power A by $\frac{1}{2}$ would produce the off-diagonal entries of the (symmetric) dioptric power matrix **F**.

It was asserted previously[1] that 'the missing $\frac{1}{2}$ in [Le Grand's] expression for the off-diagonal elements of the dioptric power matri[x] … is almost certainly a typographical error'. I am now not convinced. Although typographical errors in the 1945 edition were corrected in the 1964 edition (e.g. a missing opening bracket in the equation on the 11th line from the bottom of page 328 of Reference 2, and see Footnotes 13 and 83 of Reference 1), the missing $\frac{1}{2}$, a more important error, was not corrected. Neither was it corrected in the Spanish translation[5] that was published in 1991 nor in an unpublished list of errata (unpublished list, kindly supplied by Noberto López Gil) that appeared later.

Although given as first author, Le Grand presumably had no direct involvement in the Spanish translation (he died in 1986 [6]) but he certainly did have in the 1964 edition for, in 1980, he wrote[7] '… for the third [1964] edition I made numerous changes and additions'. Had he been aware of the missing $\frac{1}{2}$, one would have expected him to have made the correction in the 1964 edition; or, later, he might have made the error known to others and the correct formula might then have appeared in the Spanish edition or its list of errata. The fact that the error of 1945 appears never to have been corrected, despite opportunities for it to have been done and when other errors were corrected, does suggest that Le Grand did not regard it as an error. Be that as it may, there appears to be no evidence to the contrary.

One would expect to be able to find evidence for the missing factor $\frac{1}{2}$ as a typographical error in Le Grand's numerical work. Unfortunately, however, for the only case in which the formula for A is used (see page 328 of Reference 2), his power A is zero and the presence or absence of the factor makes no difference.

Had Le Grand thought in terms of a dioptric power matrix, one might have expected him to have made mention of important formulae involving **F**, including the matrix generalisations of Prentice's equation

$$\mathbf{p} = -\mathbf{F}\mathbf{y}\,, \qquad \text{[Eqn 2]}$$

Gauss's equation

$$\mathbf{L}_0 + \mathbf{F} = \mathbf{L}\,, \qquad \text{[Eqn 3]}$$

also known as the imaging equation, the equation for the power of a thin bitoric lens

$$\mathbf{F}_1 + \mathbf{F}_2 = \mathbf{F} \qquad \text{[Eqn 4]}$$

or Gullstrand's equation for a thick bitoric lens

$$\mathbf{F}_1 + \mathbf{F}_2 - \tau\,\mathbf{F}_2\mathbf{F}_1 = \mathbf{F}\,. \qquad \text{[Eqn 5]}$$

p is the prismatic effect vector at a point with position vector $\mathbf{y}$ relative to the optical centre on a thin lens of power **F**; $\mathbf{L}_0$ and **L** are incident and emergent reduced vergences; $\mathbf{F}_1$ and $\mathbf{F}_2$ are surface powers of a lens that is thin in Equation 4 and has reduced thickness τ in Equation 5. (Information on these basic formulae of linear optics is summarised elsewhere.[8]) Le Grand mentions Gullstrand's equation (see Footnotes 35 and 88 in Reference 1) but only the scalar version (i.e. the ungeneralised version). He also gives what amounts to the scalar version of Gauss's equation (the unnumbered equation at the top of page 324 of Reference 2). That he gives scalar versions without mentioning, as would seem natural and appropriate, that they generalise to these corresponding matrix versions, seems to add weight to the argument presented here.

Consequently, the weight of evidence remains in favour of the assertion that Le Grand did not describe the dioptric power matrix though he did describe aspects of it, namely the two diagonal elements f_{11} and f_{22} (Equation 1) in the case of a thin system. His expression for the off-diagonal elements was double the correct value; the missing $\frac{1}{2}$, contrary to what was asserted before[1], does not seem to have been a typographical error. Though Le Grand's four powers (three for a thin system) were elements in his matrix **M**, he did not seem to recognise those elements as constituting an important and useful submatrix (**F**) as an integral property in its own right. And he did not hint at matrix counterparts, such as Equations 2 to 5, of important well-known equations of Gaussian optics.

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Competing interests

The author declares that he has no financial or personal relationship(s) that may have inappropriately influenced him in writing this article.

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